Inflation From Symmetry Breaking Below the Planck Scale

William H. Kinney* and K.T. Mahanthappa†

Dept of Physics, Campus Box 390

University of Colorado, Boulder CO 80309-0390

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Abstract

We investigate general scalar field potentials $V(\phi)$ for inflationary cosmology arising from spontaneous symmetry breaking. We find that potentials which are dominated by terms of order ϕ^m with $m > 2$ can satisfy observational constraints at an arbitrary symmetry breaking scale. Of particular interest, the spectral index of density fluctuations is shown to be independent of the specific form of the potential, depending only on the order m of the lowest non-vanishing derivative of $V(\phi)$ near its maximum. The results of a model with a broken SO(3) symmetry illustrate these features.

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*kinney@colorado.edu

†ktm@verb.colorado.edu

Scalar field potentials arising from spontaneous symmetry breaking can in general be characterized by the presence of a "false" vacuum, an unstable or metastable equilibrium with nonzero vacuum energy density, and a physical vacuum, for which the vacuum expectation value of the scalar field ϕ is nonzero. At the physical vacuum, the potential has a stable minimum where the vacuum energy density is defined to vanish. Take a potential $V(\phi)$ described by a symmetry breaking scale v and a vacuum energy density Λ^4:

$$V(\phi) = \Lambda^4 f\left(\frac{\phi}{v}\right). \tag{1}$$

We restrict ourselves to potentials for "new" inflation, where the false vacuum is an unstable equilibrium, and inflation takes place during a period of *slow-roll*, in which the acceleration of the scalar field ϕ is negligible, $\ddot{\phi} \sim 0$. We take the first derivative of the potential to be zero at the origin and the minimum to be at $\phi = \phi_{min} \sim v$, and the function f satisfies $f'(0) = f'(\phi_{min}) = 0,\ f(0) = 1,\ f(\phi_{min}) = 0$.

A scalar field initially at some value ϕ near the false vacuum evolves to the minimum of the potential, where it oscillates and decays into other particles (*reheating*). Inflation ends and reheating commences at a field value ϕ_f, where the first order parameter $\epsilon(\phi_f)$ is unity [1]:

$$\begin{aligned}\epsilon(\phi_f) &\equiv \frac{m_{Pl}^2}{16\pi}\left(\frac{V'(\phi_f)}{V(\phi_f)}\right)^2 \equiv 1\\ &= \frac{1}{16\pi}\left(\frac{m_{Pl}}{v}\right)^2\left(\frac{f'(\phi_f/v)}{f(\phi_f/v)}\right)^2,\end{aligned} \tag{2}$$

where $m_{Pl} \sim 10^{19}\,\mathrm{GeV}$ is the Planck scale. An inflationary phase is characterized by $\epsilon < 1$: here $\epsilon(\phi = 0) = 0$ by construction. If $\epsilon(\phi)$ is everywhere increasing on the range $0 \leq \phi < \phi_{min}$, there is a unique field value ϕ_f at which inflation ends, where $\epsilon(\phi_f) \equiv 1$ and $\epsilon(\phi < \phi_f) < 1$. We are particularly interested in cases where the symmetry breaking takes place well below the Planck scale, $v \ll m_{Pl}$. Noting from (2) that $\epsilon \propto (m_{Pl}/v)^2$, the field value ϕ_f at which inflation ends is small for $v \ll m_{Pl}$, and we need only consider the behavior of the potential near the origin. We can Taylor expand $V(\phi)$ about the origin:

$$V(\phi) = V(0) + \frac{1}{m!}\frac{d^m V}{d\phi^m}\bigg|_{\phi=0}\phi^m + \cdots, \tag{3}$$

where $V'(0) \equiv 0$, and m is the order of the lowest non-vanishing derivative at the origin. For cases in which the origin is a maximum of the potential, m must be even, and $d^m V/d\phi^m < 0$. For m odd, the origin is at a saddle point, and we can define the positive ϕ direction to be such that $d^m V/d\phi^m < 0$. Models dominated by terms of order $m = 2$ are frequently discussed in the literature. Here we consider potentials for which the second derivative vanishes at the origin, $m > 2$. It is to be expected that for most potentials arising from spontaneous symmetry breaking, inflation will take place near an unstable maximum and m will be even, but this is an unnecessarily strict condition for the purpose of a general analysis. The potential can be written in the form

$$V(\phi) = \Lambda^4 \left[1 - \frac{1}{m}\left(\frac{\phi}{\mu}\right)^m + \cdots\right], \tag{4}$$

so that for $(\phi/\mu) \ll 1$, the potential is dominated by terms of order $(\phi/\mu)^m$. The vacuum energy density is $\Lambda^4 \equiv V(0)$, and $\mu \propto v$ is an effective symmetry breaking scale defined by

$$\mu \equiv \left.\left(\frac{(m-1)!V(\phi)}{|d^m V/d\phi^m|}\right)^{1/m}\right|_{\phi=0} = v\left.\left(\frac{(m-1)!f(x)}{|d^m f/dx^m|}\right)^{1/m}\right|_{x=0}. \tag{5}$$

The first order inflationary parameter ϵ is given by

$$\begin{aligned}\epsilon(\phi) &= \frac{1}{16\pi}\left(\frac{m_{Pl}}{\mu}\right)^2 \left(\frac{(\phi/\mu)^{m-1}}{1-(1/m)(\phi/\mu)^m}\right)^2 \\ &\simeq \frac{1}{16\pi}\left(\frac{m_{Pl}}{\mu}\right)^2 \left(\frac{\phi}{\mu}\right)^{2(m-1)}.\end{aligned} \tag{6}$$

Taking $\epsilon(\phi_f) \equiv 1$, we have for ϕ_f

$$\left(\frac{\phi_f}{\mu}\right) = \left[\sqrt{16\pi}\left(\frac{\mu}{m_{Pl}}\right)\right]^{1/(m-1)}. \tag{7}$$

The number of e-folds of inflation which occur when the field evolves from ϕ to ϕ_f is [2]

$$\begin{aligned}N(\phi) &= \frac{8\pi}{m_{Pl}^2}\int_{\phi_f}^{\phi} \frac{V(\phi')}{V'(\phi')}\,d\phi' \\ &= -8\pi\left(\frac{\mu}{m_{Pl}}\right)^2 \int_{\phi_f/\mu}^{\phi/\mu} \frac{1 - x^m/m}{x^{m-1}}dx \\ &\simeq 8\pi\left(\frac{\mu}{m_{Pl}}\right)^2 \left(\frac{1}{m-2}\right)\left[\left(\frac{\mu}{\phi}\right)^{m-2} - \left(\frac{\mu}{\phi_f}\right)^{m-2}\right].\end{aligned} \tag{8}$$

Smoothness on scales comparable to the current horizon size requires $N \geq 60$, which places an upper limit on the initial field value $\phi \leq \phi_{60}$, where $N(\phi_{60}) \equiv 60$. Substituting (7) for ϕ_f, we then have for ϕ_{60},

$$\left(\frac{\phi_{60}}{\mu}\right) = \left\{\frac{15(m-2)}{2\pi}\left(\frac{m_{Pl}}{\mu}\right)^2 + \left[\frac{1}{\sqrt{16\pi}}\left(\frac{m_{Pl}}{\mu}\right)\right]^{(m-2)/(m-1)}\right\}^{-1/(m-2)} . \tag{9}$$

Since $(m-2)/(m-1) < 1$, the $(m_{Pl}/\mu)^2$ term dominates, and we have the result that ϕ_{60} is to a good approximation *independent* of ϕ_f for $\mu \ll m_{Pl}$:

$$\left(\frac{\phi_{60}}{\mu}\right) \simeq \left[\frac{2\pi}{15(m-2)}\left(\frac{\mu}{m_{Pl}}\right)^2\right]^{1/(m-2)} . \tag{10}$$

This independence will be of importance when we consider the consistency of the slow-roll approximation. Quantum fluctuations in the inflaton field produce density fluctuations on scales of current astrophysical interest when $\phi \sim \phi_{60}$. The scalar density fluctuation amplitude produced during inflation is given by:

$$\begin{aligned} \delta &= \sqrt{\frac{2}{\pi}} \frac{\left[V(\phi_{60})\right]^{3/2}}{m_{Pl}^3 V'(\phi_{60})} \\ &= \sqrt{\frac{2}{\pi}} \left(\frac{\Lambda^2 \mu}{m_{Pl}^3}\right) \frac{\left[1 - (1/m)(\phi_{60}/\mu)^m\right]^{(3/2)}}{(\phi_{60}/\mu)^{m-1}} \\ &\simeq \sqrt{\frac{2}{\pi}} \left(\frac{\mu}{m_{Pl}}\right)^3 \left(\frac{\Lambda}{\mu}\right)^2 \left(\frac{\mu}{\phi_{60}}\right)^{m-1} . \end{aligned} \tag{11}$$

Substituting ϕ_{60} from (10), we have

$$\delta = \sqrt{\frac{2}{\pi}} \left(\frac{15(m-2)}{2\pi}\right)^{(m-1)/(m-2)} \left(\frac{\Lambda}{\mu}\right)^2 \left(\frac{\mu}{m_{Pl}}\right)^{(m-4)/(m-2)} . \tag{12}$$

For the case $m = 4$, the density fluctuation amplitude is independent of (μ/m_{Pl}). For $m > 4$, δ decreases with decreasing (μ/m_{Pl}) – production of density fluctuations is suppressed at low scale. We can constrain the scale Λ by using the Cosmic Background Explorer (COBE) measurement, $\delta \simeq 10^{-5}$ [3,4]:

$$\left(\frac{\Lambda}{\mu}\right)^2 = \delta \sqrt{\frac{\pi}{2}} \left(\frac{2\pi}{15(m-2)}\right)^{(m-1)/(m-2)} \left(\frac{m_{Pl}}{\mu}\right)^{(m-4)/(m-2)} , \tag{13}$$

and the constraint requires no fine-tuning of constants. Inflation is consistent only if ϕ_{60} is greater than the magnitude of quantum fluctuations on the scale of the horizon size ϕ_q, where

$$
\begin{aligned}
\left(\frac{\phi_q}{\mu}\right) &= \frac{1}{2\pi\mu}\sqrt{\frac{8\pi}{3m_{Pl}^2}V\left(0\right)} \\
&= \sqrt{\frac{2}{3\pi}}\left(\frac{\mu}{m_{Pl}}\right)\left(\frac{\Lambda}{\mu}\right)^2 \\
&= \frac{\delta}{\sqrt{3}}\left(\frac{2\pi}{15(m-2)}\right)^{(m-1)/(m-2)}\left(\frac{\mu}{m_{Pl}}\right)^{2/(m-2)},
\end{aligned} \tag{14}
$$

and the consistency condition $(\phi_q/\phi_{60}) < 1$ is satisfied independent of (μ/m_{Pl}):

$$
\left(\frac{\phi_q}{\phi_{60}}\right) = \frac{2\pi\delta}{15\sqrt{3}(m-2)}. \tag{15}
$$

The spectral index of density fluctuations, n_s, is given in terms of the slow-roll parameters ϵ and η [5]:

$$
n_s \simeq 1 - 4\epsilon\left(\phi_{60}\right) + 2\eta\left(\phi_{60}\right), \tag{16}
$$

where the second order slow-roll parameter η is defined to be:

$$
\eta\left(\phi\right) \equiv \frac{m_{Pl}^2}{8\pi}\left[\frac{V''\left(\phi\right)}{V\left(\phi\right)} - \frac{1}{2}\left(\frac{V'\left(\phi\right)}{V\left(\phi\right)}\right)^2\right]. \tag{17}
$$

For the potential (4), $V'\left(\phi_{60}\right) \sim 0$, and

$$
\begin{aligned}
n_s &\simeq 1 + \frac{m_{Pl}^2}{4\pi}\frac{V''\left(\phi_{60}\right)}{V\left(\phi_{60}\right)} \\
&\simeq 1 - \frac{m-1}{4\pi}\left(\frac{m_{Pl}}{\mu}\right)^2\left(\frac{\phi_{60}}{\mu}\right)^{m-2} \\
&= 1 - \left(\frac{1}{30}\right)\frac{m-1}{m-2},
\end{aligned} \tag{18}
$$

and we have the rather surprising result that for any $m > 2$, the scalar spectral index is independent of any characteristic of the potential except the order of the lowest non-vanishing derivative at the origin. The scalar spectral index is nearly scale invariant, with $0.93 < n_s < 0.97$ for all values of m. Thus we have the result that potentials characterized by $m > 2$ can naturally satisfy observational constraints for *any* effective symmetry breaking scale μ, where μ is proportional to the vacuum expectation value of the scalar field ϕ. The case $m = 4$ is particularly well-behaved, with both δ and n_s independent of (μ/m_{Pl}) – the Planck scale drops out of the constraints altogether.

One apparent difficulty with this class of potentials, however, is that the second order slow-roll parameter $|\eta|$ becomes large for $\phi \ll \phi_f$, so that the slow-roll approximation is invalid over much of the range at which inflation is taking place. Inflation ends at ϕ_f given by (7), but *slow-roll* ends at

$$\begin{aligned} |\eta\left(\phi\right)| &\simeq \frac{m_{Pl}^2}{8\pi}\left|\frac{V''\left(\phi\right)}{V\left(\phi\right)}\right| = 1, \\ \left(\frac{\phi}{\mu}\right) &= \left[\frac{8\pi}{m-1}\left(\frac{\mu}{m_{Pl}}\right)^2\right]^{1/(m-2)} \ll \left(\frac{\phi_f}{\mu}\right). \end{aligned} \tag{19}$$

However, from in equation (10), ϕ_{60} is *independent* of ϕ_f, so that the breakdown of slow-roll has no effect, as long as slow-roll is valid at the initial field value, $|\eta\left(\phi_{60}\right)| < 1$. If we define ϕ_{60} to be 60 e-folds before the end of slow-roll as defined in (19), instead of the end of inflation proper, we have, using (8) for $N\left(\phi\right)$,

$$\left(\frac{\phi_{60}}{\mu}\right) \simeq \left[\frac{2\pi}{15(m-2)}\left(\frac{\mu}{m_{Pl}}\right)^2\right]^{1/(m-2)}\left(1 + \frac{m-1}{60(m-2)}\right)^{-1/(m-2)}, \tag{20}$$

which is a small correction to equation (10).

Inflationary potentials characterized by $m > 2$ can arise in physically well motivated contexts. The "natural inflation" scenario [6], in which inflation is driven by a pseudo Nambu-Goldstone boson, is one such case. It can be shown that in a Lagrangian with a broken SO(3) symmetry, gauge loop effects can generate an effective potential for a Nambu-Goldstone mode θ dominated by terms of order θ^4 for small θ [7]:

$$V\left(\theta\right) = \frac{3v^4}{64\pi^2}g^4\left\{\sin^4\left(\frac{\theta}{v}\right)\ln\left[g^2\sin^2\left(\frac{\theta}{v}\right)\right] - \ln\left(g^2\right)\right\}, \tag{21}$$

where $g < 1$ is a gauge coupling constant. Note that this potential does not have a well-defined Taylor expansion at the origin, since the fourth derivative has a logarithmic (infared) divergence as $\theta \to 0$. However, the results derived here for the general potential are valid to corrections of order $\ln\left(v/m_{Pl}\right)$, and form a lowest order approximation to the exact results. In particular, the expression (18) for the spectral index of scalar density fluctuations is exact, with $n_s = 0.95$. A lower bound on the symmetry breaking scale v can be obtained with the inclusion of fermions. A detailed analysis will be presented in a forthcoming paper [7].

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